ON ESTIMATING THE INFLUENCE OF THE TERMS CHARACTERIZING THE CHANGE IN VELOCITY HEAD IN THE EQUATIONS GOVERNING THE TRANSIENT NONISOTHERMAL MOTION OF GAS IN PIPELINES

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A system of equations describing the transient, nonisothermal flow of gas in pipelines is considered, and the effect of the terms characterizing the change in the velocity head on the character of the solution is analyzed. A comparison is drawn between the numerical solution of these equations with and without allowing for the changes in the velocity head, and some corresponding estimates of the differences between these are presented.

According to [1] the differential equations representing the transient, nonisothermal flow of gas in a horizontal pipe may be expressed in the following manner (assuming that the temperature of the external medium is a known function of the coordinate, or a constant quantity, and that heat transfer obeys Newton's law, while the transfer of heat along the axis of the pipe by conduction in the gas may be neglected):

$$\frac{\partial \rho}{\partial t}+\frac{\partial (\rho w)}{\partial x}=0, \tag{1}$$

$$\frac{\partial (\rho w)}{\partial t}+\frac{\partial}{\partial x}\,(P+\rho w^2)=-\rho\,\frac{\lambda}{2D}\,w\,|w|, \tag{2}$$

$$\frac{\partial}{\partial t}\left[\rho\left(\frac{U}{A}+\frac{w^2}{2}\right)\right]+\frac{\partial}{\partial x}\left[\rho w\left(\frac{U}{A}+\frac{P}{\rho}+\frac{w^2}{2}\right)\right]=\frac{4}{D}\,\frac{K(T_0-T)}{A}\,. \tag{3}$$

In order to complete the system of equations (1)-(3) we add the equation of state of the gas in the form

$$\frac{P}{\gamma}=z_0(P,\ T)\,RT,\qquad 0<x<L,\quad t>0. \tag{4}$$

It was pointed out by I. A. Charnyi [2] that, in the case of transient gas flows in long tubes (gas pipelines) taking place at velocities considerably below the velocity of sound, the changes in velocity head i.e., the terms $w(\partial w/\partial x)$ and $\partial w/\partial t$, which only exert a very small influence under these circumstances, might be neglected in the equation of motion. However, our own study of the literature has not revealed any accurate estimates to justify this in the case of the transient, nonisothermal gas flows described by Eqs. (1)-(4).

We shall now compare some numerical solutions of Eqs. (1)-(4), carried out with and without allowing for changes in the velocity head, in relation to the flow of gas when the end of a pipeline is instantaneously covered. This constitutes a limiting case, providing a fair estimate of the maximum possible deviations.

It was shown in [3, 4] that, by neglecting the terms $w(\partial w/\partial x)$ and $\partial w/\partial t$, Eqs. (1)-(4) could be reduced to the following system of quasilinear differential equations of the evolution type;

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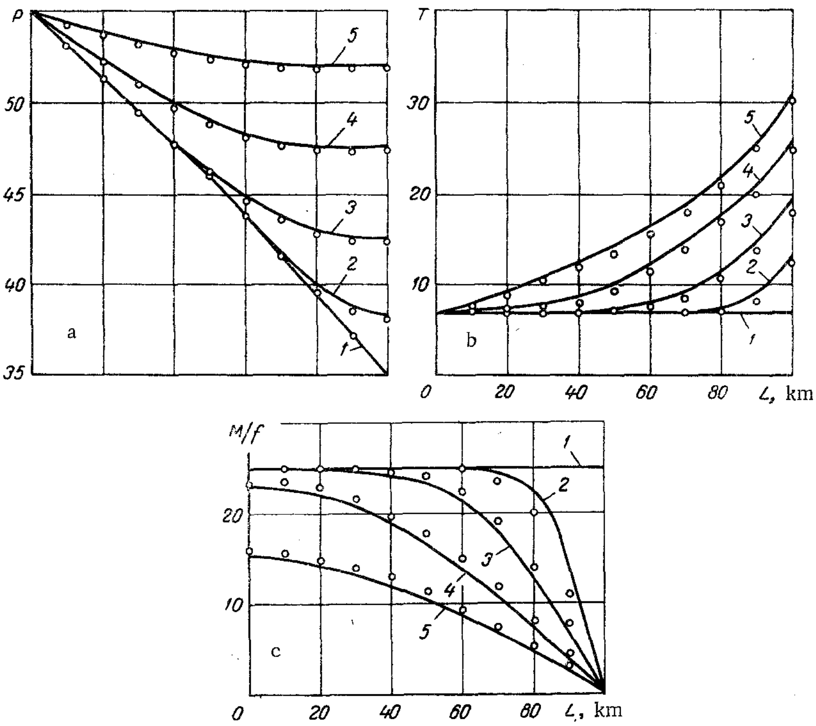

Fig. 1. Change in pressure (a), temperature (b), and mass flow of gas (c) along the gas pipeline with time: 1) t = 0; 2) 1; 3) 5; 4) 15; 5) 30 min; curves 1-5, without allowing for the inertial terms; points, allowing for the inertial terms. P, kg/cm^2; T, °C; M/f, $kg \cdot sec/m^3$; L, km.

$$\frac{\partial P}{\partial t}=\left\{\frac{1}{2a}\left[\frac{1}{2bG}\cdot\frac{\partial^2 P^2}{\partial x^2}+G\frac{\partial}{\partial x}(z_0T)\right]-\left[z_0+T\left(\frac{\partial z_0}{\partial T}\right)_P\right]\right.$$
$$\left.\times\left[\frac{1}{a}G\frac{\partial T}{\partial x}+bc\frac{G^3T^3z_0}{P^2}\left(\frac{\partial z_0}{\partial T}\right)_P-n^*(T_0-T)\right]\right\}\Big/\Delta, \tag{5}$$

$$\frac{\partial T}{\partial t}=\frac{T}{P}\cdot\left\{\frac{1}{2a}\left[\frac{1}{2bG}\cdot\frac{\partial^2 P^2}{\partial x^2}+G\frac{\partial}{\partial x}(z_0T)\right]\right.$$
$$\left.\times\left[z_0+T\left(\frac{\partial z_0}{\partial T}\right)_P\right]m+z_0\left[\frac{1}{a}G\frac{\partial T}{\partial x}+bc\frac{G^3T^3z_0}{P^2}\left(\frac{\partial z_0}{\partial T}\right)_P-n^*(T_0-T)\right]\left(\frac{P}{z_0}\cdot\frac{\partial z_0}{\partial P}-1\right)\right\}\Big/\Delta, \tag{6}$$

$$\frac{\partial P^2}{\partial x}=-2bG^2z_0T, \tag{7}$$

$$0<x<L,\ t>0,$$

where

$$\Delta\equiv 1-\frac{P}{z_0}\left(\frac{\partial z_0}{\partial P}\right)_T-\frac{m}{z_0}\left[z_0+T\left(\frac{\partial z_0}{\partial T}\right)_P\right]^2,\ z_0=z_0(P,\ T),$$

$$a=\frac{f}{R},\ b=\frac{\lambda R}{2gDf^2},\ c=\frac{\lambda AR^3}{2gDf^3c_p},\ m=\frac{AR}{c_p},\ n^*=\frac{K\pi DR}{c_pf}.$$

For the numerical solution of Eqs. (5)-(7) by the method of finite differences, finite-difference schemes of the explicit type were proposed in [4, 5].

The use of finite-difference schemes involving asymmetric difference equations was considered in [6], as well as modifications of these based on the alternating method and the method of the arithmetic mean.

In the case of the transient, nonisothermal flow of gas arising from the instantaneous closing of the end of the pipeline considered below, the boundary conditions take the form

$$P(0,\ t)=f_1(t),\ T(0,\ t)=f_2(t),\ \frac{M}{f}(L,\ t)=\rho w(L,\ t)=0. \tag{8}$$

As the initial distribution of the unknown quantities, we take the flow parameters obtained for a steady, nonisothermal flow.*

For the numerical solution of the system of equations (5)-(6), with the initial and boundary conditions indicated (8), in contrast to the schemes of the first order of accuracy which were presented in [4, 5], we employed an explicit finite-difference scheme of the second order of accuracy with respect to the step h. The derivatives with respect to x at the internal points were approximated by means of the symmetrical difference relationships

$$\left(\frac{\partial^2 P^2}{\partial x^2}\right)_{i,k} = \frac{P^2_{i-1,k} - 2P^2_{i,k} + P^2_{i+1,k}}{h^2} + O(h^2),$$

$$\left(\frac{\partial T}{\partial x}\right)_{i,k} = \frac{T_{i+1,k} - T_{i-1,k}}{2h} + O(h^2), \quad \left(\frac{\partial P}{\partial x}\right)_{i,k} = \frac{P_{i+1,k} - P_{i-1,k}}{2h} + O(h^2)$$

$$(i=1,\ 2,\ \ldots,\ n-1;\ k=1,\ 2,\ \ldots;\ n=1/h).$$

For an approximation of the derivatives at the boundary points we used the corresponding unilateral three-point difference formulas.

In order to calculate the values of G we used the following finite-difference formulas:

$$G_{i,k} = -\operatorname{sign}(P_{i+1,k} - P_{i-1,k})\left[\frac{1}{2b(z_0T)_{i,k}}\left|\frac{P^2_{i+1,k} - P^2_{i-1,k}}{2h}\right|\right]^{1/2} \tag{9}$$

$$(i=1,\ 2,\ \ldots,\ n-1;\ k=1,\ 2,\ 3,\ \ldots),$$

$$G_{0,k} = -\operatorname{sign}(4P_{1,k} - 3P_{0,k} - P_{2,k})\left[\frac{1}{2b(z_0T)_{0,k}}\left|\frac{4P^2_{1,k} - 3P^2_{0,k} - P^2_{2,k}}{2h}\right|\right]^{1/2}. \tag{10}$$

In the present case governed by the boundary conditions (8), in order to determine the temperature values T at the points of the right-hand boundary ($x = L$) we used finite-difference formulas of the kind given in [5].

By way of example, Figure 1 shows the distribution of P, T, and M/f respectively along the pipe for an ideal gas at various instants of time, obtained by computer calculation (the continuous lines correspond to the distribution without allowing for the velocity head and the points to the inclusion of the latter).

The distribution in question corresponds to the boundary conditions

$$P(0,\ t)=55\cdot 10^4\ \text{kg/m}^2;\ T(0,\ t)=7\,^\circ\text{C};\ \frac{M}{f}(L,\ t)=0 \tag{11}$$

and to an initial distribution of flow parameters obtained for steady, isothermal conditions with

$$P(0,\ 0)=55\cdot 10^4\ \frac{\text{kg}}{\text{m}^2};\ T(0,\ 0)=7\,^\circ\text{C};\ \frac{M}{f}(0,\ 0)=25\ \text{kg}\cdot\text{sec/m}^3. \tag{12}$$

The values of the parameters entering into Eqs. (1)-(4) and (5)-(7) were taken as follows: L = 100 km; D = 0.7 m; R = 53 kg · m/kg · °K; c_p = 0.55 kcal/kg · deg; λ = 0.01386; K = 1 kcal/h · m^2 · °C; T_0 = 280°K.

It follows from the results obtained in Fig. 1 that the difference between the two solutions is no greater than 1% as regards temperature and pressure.†

The difference in flow rates (Fig. 1c) at the initial instant of time equals 20% (at points close to the closed end), which is quite natural, since the inertial terms were neglected.

In practice, the solutions of Eqs. (1)-(4) with and without considering the change in the velocity head agree to an accuracy of 4-5% for a t of the order of 30 min. This approximately corresponds to six times the running time of a gas pressure wave at the velocity of sound, in agreement with the estimate of I. A. Charnyi [2].

*The determination of the parameters of this kind of flow involves the solution of a Cauchy problem for the corresponding system of nonlinear ordinary differential equations, subject to specified conditions regarding the pressure and temperature at one end of the pipe [4, 5].

†In the example considered, the numerical solution of Eqs. (5)-(7) obtained by the authors was compared with the numerical solution of Eqs. (1)-(4) obtained by the method of finite differences [7] for the foregoing initial and boundary conditions by O. F. Vasil'ev and A. F. Voevodin.

TABLE 1. Comparison between the Distribution of the Parameters of a Transient Gas Flow along a Gas Pipeline for Real and Ideal Gases in the Nonisothermal Mode

t	x	P		T		$\frac{M}{f}$	
		I	II	I	II	I	II
	0	55,0	55,0	47,0	47,0	25,0	25,0
	10	53,218	53,109	41,971	42,559	25,0	25,0
	20	51,407	51,174	37,469	38,612	25,0	25,0
	30	49,559	49,188	33,434	35,102	25,0	25,0
	40	47,665	47,141	29,811	31,982	25,0	25,0
0	50	45,718	45,022	26,551	29,209	25,0	25,0
	60	43,705	42,817	23,611	26,743	25,0	25,0
	70	41,615	40,511	20,950	24,552	25,0	25,0
	80	39,431	38,083	18,530	22,603	25,0	25,0
	90	37,134	35,507	16,317	20,871	25,0	25,0
	100	34,699	32,744	14,277	19,331	25,0	25,0
	0	55,0	55,0	47,0	47,0	24,348	24,547
	10	53,319	53,185	42,109	42,642	24,184	24,415
	20	51,657	51,369	37,836	38,841	23,706	24,013
	30	50,060	49,599	34,190	35,599	22,799	23,222
	40	48,594	47,941	31,240	32,974	21,320	21,873
10	50	47,334	46,486	29,093	31,069	19,141	19,799
	60	46,354	45,329	27,871	30,021	16,214	16,901
	70	45,690	44,531	27,671	29,971	12,623	13,236
	80	45,325	44,088	28,554	31,042	8,571	9,020
	90	45,187	43,919	30,573	33,403	4,299	4,530
	100	45,168	43,896	33,941	37,542	0	0
	0	55,0	55,0	47,0	47,0	22,655	23,161
	10	53,554	53,394	42,367	42,815	22,366	22,906
	20	52,158	51,824	38,492	39,308	21,625	22,218
	30	50,867	50,352	35,300	36,417	20,379	21,030
	40	49,740	49,047	32,851	34,200	18,595	19,285
15	50	48,825	47,972	31,210	32,730	16,278	16,966
	60	48,149	47,167	30,430	32,082	13,488	14,118
	70	47,711	46,641	30,551	32,329	10,337	10,852
	80	47,477	46,357	31,608	33,566	6,959	7,318
	90	47,389	46,251	33,689	35,991	3,482	3,663
	100	47,378	46,237	37,084	40,184	0	0
	0	55,0	55,0	47,0	47,0	20,381	21,147
	10	53,837	53,668	42,567	42,942	20,035	20,830
	20	52,732	52,388	39,125	39,778	19,219	20,038
	30	51,733	51,218	36,356	37,228	17,931	18,753
	40	50,884	50,211	34,300	35,335	16,185	16,980
20	50	50,211	49,405	32,993	34,147	14,026	14,755
	60	49,725	48,819	32,464	33,714	11,529	12,155
	70	49,415	48,443	32,740	34,095	8,789	9,280
	80	49,251	48,243	33,867	35,389	5,902	6,236
	90	49,190	48,169	35,958	37,820	2,952	3,119
	100	49,182	48,159	39,352	42,028	0	0
	0	55,0	55,0	47,0	47,0	15,370	16,442
	10	54,346	54,203	42,515	42,791	15,010	16,111
	20	53,740	53,455	39,807	40,244	14,251	15,355
	30	53,208	52,790	37,709	38,260	13,161	14,224
	40	52,766	52,233	36,219	36,845	11,770	12,752
30	50	52,425	51,799	35,349	36,024	10,123	10,987
	60	52,183	51,490	35,116	35,833	8,278	8,994
	70	52,030	51,295	35,552	36,335	6,293	6,841
	80	51,949	51,192	36,725	37,651	4,224	4,592
	90	51,919	51,154	38,787	40,040	2,115	2,299
	100	51,915	51,148	42,129	44,208	0	0

Note: I) real gas; II) ideal gas; the quantities given in the table have the following dimensions: t) min; x) km; P) kg/cm^2; T) °C; M/f) kg·sec/m^3.

It is interesting to note that, at the onset of the process ($t \leq 1$ min), the solution obtained without allowing for the change in the velocity head gives too high a result, while for long periods the result is too low.

Table 1 represents a comparison between the solutions of Eqs. (5)-(7) with the same values of the initial parameters for a real and an ideal gas, obtained by computer calculation subject to the boundary conditions

$$P(0,\ t)=55\cdot 10^4\ \mathrm{kg/m^2};\ T(0,\ t)=47\ ^\circ\mathrm{C};\ \frac{M}{f}\ (L,\ t)=0. \tag{13}$$

The initial distribution of flow parameters was obtained for a steady-state nonisothermal flow by the Runge–Kutta method with

$$P(0,\ 0)=55\cdot 10^4\ \frac{\mathrm{kg}}{\mathrm{m^2}};\ T(0,\ 0)=47\ ^\circ\mathrm{C};\ \frac{M}{f}\ (0,\ 0)=25\ \mathrm{kg\cdot sec/m^3}. \tag{14}$$

In calculating the flow parameters of a real gas, we used the equation of state of a real gas in the Berthelot form.

We see from Table 1 that the pressure at the end of the tube is higher for the real than for the ideal gas, by about 6% at $t = 0$, while the temperature of the real gas is lower than that of the ideal (by 5.2°C or 1.7% at $t = 0$). With increasing time the difference between the temperatures of the real and ideal gases diminishes.

The mass flow of the real gas is about 4-5% lower than that of the ideal for $t > 0$.

Thus our numerical calculations have shown that, in practice, for periods of greater than 30 min, the effect of the terms characterizing the change in the velocity head on the distribution of the parameters of a transient, nonisothermal gas flow (as encountered in the cases of gas motion discussed here) is negligible; these terms may indeed be omitted without detriment to practical accuracy when calculating the distribution of temperature, pressure, and mass flow for gas in a pipeline.

NOTATION

P, ρ, w	are the pressure, density, and velocity of the gas averaged over the cross section of the pipe, respectively;
γ	is the specific gravity of the gas;
U	is the internal energy;
A	is the heat equivalent of mechanical work;
λ	is the hydraulic resistance;
T, T_0	are the temperature of the gas and the external medium respectively;
G	is the gravimetric flow of gas;
M	is the mass flow of gas;
f, D	are the cross-sectional area and diameter of the pipe, respectively;
x	is the coordinate along the axis of the pipe;
t	is the time;
h	is the step in the coordinate x.

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